Bosnia Herzegovina
Team Selection Test
2008

Day 1

1. Prove that in an isosceles triangle $\triangle ABC$ with $AC = BC = b$ following inequality holds $b > \pi r$, where r is inradius.

2. Find all pairs of positive integers m and n that satisfy (both) following conditions:

   (i) $m^2 - n$ divides $m + n^2$

   (ii) $n^2 - m$ divides $n + m^2$

3. 30 persons are sitting at round table. $30 - N$ of them always speak true (quot;true speakersquot;) while the other N of them sometimes speak true sometimes not (quot;lie speakersquot;). Question: quot;Who is your right neighbour - quot;true speakerquot; or quot;lie speakerquot; ?quot; is asked to all 30 persons and 30 answers are collected. What is maximal number N for which (with knowledge of these answers) we can always be sure (decide) about at least one person who is quot;true speakerquot;.

   ANSWER:answer is 8

Day 2

1. 8 students took part in exam that contains 8 questions. If it is known that each question was solved by at least 5 students, prove that we can always find 2 students such that each of questions was solved by at least one of them.

2. Let AD be height of triangle $\triangle ABC$ and R circumradius. Denote by E and F feet of perpendiculars from point D to sides AB and AC.

   If $AD = R\sqrt{2}$, prove that circumcenter of triangle $\triangle ABC$ lies on line EF.

3. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(f(x) + y) = f(x^2 - y) + 4f(x)y$$

   for all $x, y \in \mathbb{R}$.