# Germany
## Bundeswettbewerb Mathematik
2008

## Round 1

1 Fedja used matches to put down the equally long sides of a parallelogram whose vertices are not on a common line. He figures out that exactly 7 or 9 matches, respectively, fit into the diagonals. How many matches compose the parallelogram's perimeter?

2 Represent the number 2008 as a sum of natural number such that the addition of the reciprocals of the summands yield 1.

3 Prove: In an acute triangle $ABC$ angle bisector $w_\alpha$, median $s_b$ and the altitude $h_c$ intersect in one point if $w_\alpha$, side $BC$ and the circle around foot of the altitude $h_c$ have vertex $A$ as a common point.

4 In a planar coordinate system we got four pieces on positions with coordinates. You can make a move according to the following rule: You can move a piece to a new position if there is one of the other pieces in the middle of the old and new position. Initially the four pieces have positions $\{(0,0),(0,1),(1,0),(1,1)\}$. Given a finite number of moves can you yield the configuration $\{(0,0),(1,1),(3,0),(2,-1)\}$ ?

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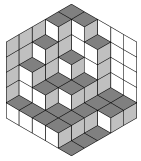

## Round 2

1 Determine all real $x$ satisfying the equation

$$\sqrt[5]{x^3+2x} = \sqrt[3]{x^5-2x}.$$

Odd roots for negative radicands shall be included in the discussion.

2 Let the positive integers $a, b, c$ chosen such that the quotients $\frac{bc}{b+c}$, $\frac{ca}{c+a}$ and $\frac{ab}{a+b}$ are integers. Prove that $a, b, c$ have a common divisor greater than 1.

3 Through a point in the interior of a sphere we put three pairwise perpendicular planes. Those planes dissect the surface of the sphere in eight curvilinear triangles. Alternately the triangles are coloured black and wide to make the sphere surface look like a checkerboard. Prove that exactly half of the sphere's surface is coloured black.

4 On a bookcase there are $n \geq 3$ books side by side by different authors. A librarian considers the first and second book from left and exchanges them iff they are not alphabetically sorted. Then he is doing the same operation with the second and third book from left etc. Using this procedure he iterates through the bookcase three times from left to right. Considering all possible initial book configurations how many of them will then be alphabetically sorted?

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