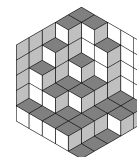




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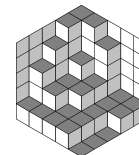


**First Round**

- 1 (B.Frenkin, 8) Does a regular polygon exist such that just half of its diagonals are parallel to its sides?
- 2 (V.Protasov, 8) For a given pair of circles, construct two concentric circles such that both are tangent to the given two. What is the number of solutions, depending on location of the circles?
- 3 (A.Zaslavsky, 8) A triangle can be dissected into three equal triangles. Prove that some its angle is equal to  $60^\circ$ .
- 4 (D.Shnol, 8–9) The bisectors of two angles in a cyclic quadrilateral are parallel. Prove that the sum of squares of some two sides in the quadrilateral equals the sum of squares of two remaining sides.
- 5 (Kiev olympiad, 8–9) Reconstruct the square  $ABCD$ , given its vertex  $A$  and distances of vertices  $B$  and  $D$  from a fixed point  $O$  in the plane.
- 6 (A. Myakishev, 8–9) In the plane, given two concentric circles with the center  $A$ . Let  $B$  be an arbitrary point on some of these circles, and  $C$  on the other one. For every triangle  $ABC$ , consider two equal circles mutually tangent at the point  $K$ , such that one of these circles is tangent to the line  $AB$  at point  $B$  and the other one is tangent to the line  $AC$  at point  $C$ . Determine the locus of points  $K$ .
- 7 (A.Zaslavsky, 8–9) Given a circle and a point  $O$  on it. Another circle with center  $O$  meets the first one at points  $P$  and  $Q$ . The point  $C$  lies on the first circle, and the lines  $CP$ ,  $CQ$  meet the second circle for the second time at points  $A$  and  $B$ . Prove that  $AB = PQ$ .
- 8 (T.Golenishcheva-Kutuzova, B.Frenkin, 8–11) a) Prove that for  $n > 4$ , any convex  $n$ -gon can be dissected into  $n$  obtuse triangles.
- 9 (A.Zaslavsky, 9–10) The reflections of diagonal  $BD$  of a quadrilateral  $ABCD$  in the bisectors of angles  $B$  and  $D$  pass through the midpoint of diagonal  $AC$ . Prove that the reflections of diagonal  $AC$  in the bisectors of angles  $A$  and  $C$  pass through the midpoint of diagonal  $BD$  (There was an error in published condition of this problem).
- 10 (A.Zaslavsky, 9–10) Quadrilateral  $ABCD$  is circumscribed around a circle with center  $I$ . Prove that the projections of points  $B$  and  $D$  to the lines  $IA$  and  $IC$  lie on a single circle.
- 11 (A.Zaslavsky, 9–10) Given four points  $A$ ,  $B$ ,  $C$ ,  $D$ . Any two circles such that one of them contains  $A$  and  $B$ , and the other one contains  $C$  and  $D$ , meet. Prove that common chords of all these pairs of circles pass through a fixed point.



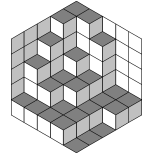
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- 12] (A.Myakishev, 9–10) Given a triangle  $ABC$ . Point  $A_1$  is chosen on the ray  $BA$  so that segments  $BA_1$  and  $BC$  are equal. Point  $A_2$  is chosen on the ray  $CA$  so that segments  $CA_2$  and  $BC$  are equal. Points  $B_1, B_2$  and  $C_1, C_2$  are chosen similarly. Prove that lines  $A_1A_2, B_1B_2, C_1C_2$  are parallel.
- 13] (A.Myakishev, 9–10) Given triangle  $ABC$ . One of its excircles is tangent to the side  $BC$  at point  $A_1$  and to the extensions of two other sides. Another excircle is tangent to side  $AC$  at point  $B_1$ . Segments  $AA_1$  and  $BB_1$  meet at point  $N$ . Point  $P$  is chosen on the ray  $AA_1$  so that  $AP = NA_1$ . Prove that  $P$  lies on the incircle.
- 14] (V.Protasov, 9–10) The Euler line of a non-isosceles triangle is parallel to the bisector of one of its angles. Determine this angle (There was an error in published condition of this problem).
- 15] (M.Volchkevich, 9–11) Given two circles and point  $P$  not lying on them. Draw a line through  $P$  which cuts chords of equal length from these circles.
- 16] (A.Zaslavsky, 9–11) Given two circles. Their common external tangent is tangent to them at points  $A$  and  $B$ . Points  $X, Y$  on these circles are such that some circle is tangent to the given two circles at these points, and in similar way (external or internal). Determine the locus of intersections of lines  $AX$  and  $BY$ .
- 17] (A.Myakishev, 9–11) Given triangle  $ABC$  and a ruler with two marked intervals equal to  $AC$  and  $BC$ . By this ruler only, find the incenter of the triangle formed by medial lines of triangle  $ABC$ .
- 18] (A.Abdullayev, 9–11) Prove that the triangle having sides  $a, b, c$  and area  $S$  satisfies the inequality
- $$a^2 + b^2 + c^2 - \frac{1}{2}(|a - b| + |b - c| + |c - a|)^2 \geq 4\sqrt{3}S.$$
- 19] (V.Protasov, 10-11) Given parallelogram  $ABCD$  such that  $AB = a, AD = b$ . The first circle has its center at vertex  $A$  and passes through  $D$ , and the second circle has its center at  $C$  and passes through  $D$ . A circle with center  $B$  meets the first circle at points  $M_1, N_1$ , and the second circle at points  $M_2, N_2$ . Determine the ratio  $M_1N_1/M_2N_2$ .
- 20] (A.Zaslavsky, 10–11) a) Some polygon has the following property: if a line passes through two points which bisect its perimeter then this line bisects the area of the polygon. Is it true that the polygon is central symmetric? b) Is it true that any figure with the property from part a) is central symmetric?
- 21] (A.Zaslavsky, B.Frenkin, 10–11) In a triangle, one has drawn perpendicular bisectors to its sides and has measured their segments lying inside the triangle.



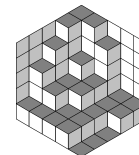
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- a) All three segments are equal. Is it true that the triangle is equilateral?
- b) Two segments are equal. Is it true that the triangle is isosceles?
- c) Can the segments have length 4, 4 and 3?
- 22 (A.Khachatryan, 10–11) a) All vertices of a pyramid lie on the facets of a cube but not on its edges, and each facet contains at least one vertex. What is the maximum possible number of the vertices of the pyramid?
- b) All vertices of a pyramid lie in the facet planes of a cube but not on the lines including its edges, and each facet plane contains at least one vertex. What is the maximum possible number of the vertices of the pyramid?
- 23 (V.Protasov, 10–11) In the space, given two intersecting spheres of different radii and a point  $A$  belonging to both spheres. Prove that there is a point  $B$  in the space with the following property: if an arbitrary circle passes through points  $A$  and  $B$  then the second points of its meet with the given spheres are equidistant from  $B$ .
- 24 (I.Bogdanov, 11) Let  $h$  be the least altitude of a tetrahedron, and  $d$  the least distance between its opposite edges. For what values of  $t$  the inequality  $d > th$  is possible?



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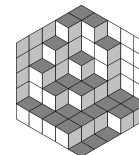


Grade 8

- 1 (B.Frenkin) Does a convex quadrilateral without parallel sidelines exist such that it can be divided into four congruent triangles?
- 2 (F.Nilov) Given right triangle  $ABC$  with hypotenuse  $AC$  and  $\angle A = 50^\circ$ . Points  $K$  and  $L$  on the cathetus  $BC$  are such that  $\angle KAC = \angle LAB = 10^\circ$ . Determine the ratio  $CK/LB$ .
- 3 (D.Shnol) Two opposite angles of a convex quadrilateral with perpendicular diagonals are equal. Prove that a circle can be inscribed in this quadrilateral.
- 4 (F.Nilov, A.Zaslavsky) Let  $CC_0$  be a median of triangle  $ABC$ ; the perpendicular bisectors to  $AC$  and  $BC$  intersect  $CC_0$  in points  $A'$ ,  $B'$ ;  $C_1$  is the meet of lines  $AA'$  and  $BB'$ . Prove that  $\angle C_1CA = \angle C_0CB$ .
- 5 (A.Zaslavsky) Given two triangles  $ABC$ ,  $A'B'C'$ . Denote by  $\alpha$  the angle between the altitude and the median from vertex  $A$  of triangle  $ABC$ . Angles  $\beta$ ,  $\gamma$ ,  $\alpha'$ ,  $\beta'$ ,  $\gamma'$  are defined similarly. It is known that  $\alpha = \alpha'$ ,  $\beta = \beta'$ ,  $\gamma = \gamma'$ . Can we conclude that the triangles are similar?
- 6 (B.Frenkin) Consider the triangles such that all their vertices are vertices of a given regular 2008-gon. What triangles are more numerous among them: acute-angled or obtuse-angled?
- 7 (F.Nilov) Given isosceles triangle  $ABC$  with base  $AC$  and  $\angle B = \alpha$ . The arc  $AC$  constructed outside the triangle has angular measure equal to  $\beta$ . Two lines passing through  $B$  divide the segment and the arc  $AC$  into three equal parts. Find the ratio  $\alpha/\beta$ .
- 8 (B.Frenkin, A.Zaslavsky) A convex quadrilateral was drawn on the blackboard. Boris marked the centers of four excircles each touching one side of the quadrilateral and the extensions of two adjacent sides. After this, Alexey erased the quadrilateral. Can Boris define its perimeter?



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**Grade 9**

- 1 (A.Zaslavsky) A convex polygon can be divided into 2008 congruent quadrilaterals. Is it true that this polygon has a center or an axis of symmetry?
- 2 (F.Nilov) Given quadrilateral  $ABCD$ . Find the locus of points such that their projections to the lines  $AB, BC, CD, DA$  form a quadrilateral with perpendicular diagonals.
- 3 (R.Pirkuliev) Prove the inequality

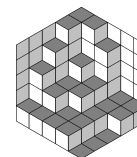
$$\frac{1}{\sqrt{2 \sin A}} + \frac{1}{\sqrt{2 \sin B}} + \frac{1}{\sqrt{2 \sin C}} \leq \sqrt{\frac{p}{r}},$$

where  $p$  and  $r$  are the semiperimeter and the inradius of triangle  $ABC$ .

- 4 (F.Nilov, A.Zaslavsky) Let  $CC_0$  be a median of triangle  $ABC$ ; the perpendicular bisectors to  $AC$  and  $BC$  intersect  $CC_0$  in points  $A_c, B_c$ ;  $C_1$  is the common point of  $AA_c$  and  $BB_c$ . Points  $A_1, B_1$  are defined similarly. Prove that circle  $A_1B_1C_1$  passes through the circumcenter of triangle  $ABC$ .
- 5 (N.Avilov) Can the surface of a regular tetrahedron be glued over with equal regular hexagons?
- 6 (B.Frenkin) Construct the triangle, given its centroid and the feet of an altitude and a bisector from the same vertex.
- 7 (A.Zaslavsky) The circumradius of triangle  $ABC$  is equal to  $R$ . Another circle with the same radius passes through the orthocenter  $H$  of this triangle and intersect its circumcircle in points  $X, Y$ . Point  $Z$  is the fourth vertex of parallelogram  $CXZY$ . Find the circumradius of triangle  $ABZ$ .
- 8 (J.-L.Ayme, France) Points  $P, Q$  lie on the circumcircle  $\omega$  of triangle  $ABC$ . The perpendicular bisector  $l$  to  $PQ$  intersects  $BC, CA, AB$  in points  $A', B', C'$ . Let  $A\text{quot}, B\text{quot}, C\text{quot}$ ; be the second common points of  $l$  with the circles  $A'PQ, B'PQ, C'PQ$ . Prove that  $AA\text{quot}, BB\text{quot}, CC\text{quot}$ ; concur.



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Grade 10

- 1 (B.Frenkin) An inscribed and circumscribed  $n$ -gon is divided by some line into two inscribed and circumscribed polygons with different numbers of sides. Find  $n$ .
- 2 (A.Myakishev) Let triangle  $A_1B_1C_1$  be symmetric to  $ABC$  wrt the incenter of its medial triangle. Prove that the orthocenter of  $A_1B_1C_1$  coincides with the circumcenter of the triangle formed by the excenters of  $ABC$ .
- 3 (V.Yasinsky, Ukraine) Suppose  $X$  and  $Y$  are the common points of two circles  $\omega_1$  and  $\omega_2$ . The third circle  $\omega$  is internally tangent to  $\omega_1$  and  $\omega_2$  in  $P$  and  $Q$  respectively. Segment  $XY$  intersects  $\omega$  in points  $M$  and  $N$ . Rays  $PM$  and  $PN$  intersect  $\omega_1$  in points  $A$  and  $D$ ; rays  $QM$  and  $QN$  intersect  $\omega_2$  in points  $B$  and  $C$  respectively. Prove that  $AB = CD$ .
- 4 (A.Zaslavsky) Given three points  $C_0, C_1, C_2$  on the line  $l$ . Find the locus of incenters of triangles  $ABC$  such that points  $A, B$  lie on  $l$  and the feet of the median, the bisector and the altitude from  $C$  coincide with  $C_0, C_1, C_2$ .
- 5 (I.Bogdanov) A section of a regular tetragonal pyramid is a regular pentagon. Find the ratio of its side to the side of the base of the pyramid.
- 6 (B.Frenkin) The product of two sides in a triangle is equal to  $8Rr$ , where  $R$  and  $r$  are the circumradius and the inradius of the triangle. Prove that the angle between these sides is less than  $60^\circ$ .
- 7 (F.Nilov) Two arcs with equal angular measure are constructed on the medians  $AA'$  and  $BB'$  of triangle  $ABC$  towards vertex  $C$ . Prove that the common chord of the respective circles passes through  $C$ .
- 8 (A.Akopyan, V.Dolnikov) Given a set of points in the plane. It is known that among any three of its points there are two such that the distance between them doesn't exceed 1. Prove that this set can be divided into three parts such that the diameter of each part does not exceed 1.