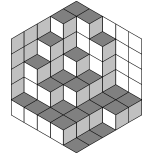




Vietnam National Olympiad 2008



- 1 Determine the number of solutions of simultaneous equations to $x^2 + y^3 = 29$ and $\log_3 x \cdot \log_2 y = 1$.
- 2 Given a triangle with acute angle $\angle BEC$, let E be the midpoint of AB . Point M is chosen on the opposite ray of EC such that $\angle BME = \angle ECA$. Denote by θ the measure of angle $\angle BEC$. Evaluate $\frac{MC}{AB}$ in terms of θ .
- 3 Let $m = 2007^{2008}$, how many natural numbers n are there such that $n < m$ and $n(2n + 1)(5n + 2)$ is divisible by m (which means that $m \mid n(2n + 1)(5n + 2)$) ?
- 4 The sequence of real number (x_n) is defined by $x_1 = 0$, $x_2 = 2$ and $x_{n+2} = 2^{-x_n} + \frac{1}{2} \forall n = 1, 2, 3, \dots$. Prove that the sequence has a limit as n approaches $+\infty$. Determine the limit.
- 5 What is the total number of natural numbers divisible by 9 the number of digits of which does not exceed 2008 and at least two of the digits are 9s?
- 6 Let x, y, z be distinct non-negative real numbers. Prove that

$$\frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} \geq \frac{4}{xy + yz + zx}.$$

When does the equality hold?

- 7 Let D is centroid of ABC triangle. Let (d) is the perpendicular line with AD . Let M is a point on (d) . Let E, F are midpoints of MB, MC respectively. The line through point E and perpendicular with (d) meet AB at P . The line through point F and perpendicular with (d) meet AC at Q . Let (d') is a line through point M and perpendicular with PQ . Prove (d') always pass a fixed point.

_____ I used geosketpad and find the point
S. Can you prove it?