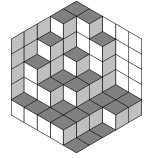




Vietnam
Team Selection Tests
2008

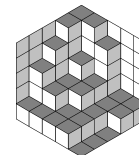


Day 1

- 1 On the plane, given an angle xOy . M be a mobile point on ray Ox and N a mobile point on ray Oy . Let d be the external angle bisector of angle xOy and I be the intersection of d with the perpendicular bisector of MN . Let P, Q be two points lie on d such that $IP = IQ = IM = IN$, and let K the intersection of MQ and NP .
1. Prove that K always lie on a fixed line.
 2. Let d_1 line perpendicular to IM at M and d_2 line perpendicular to IN at N . Assume that there exist the intersections E, F of d_1, d_2 from d . Prove that EN, FM and OK are concurrent.
- 2 Find all values of the positive integer m such that there exists polynomials with real coefficient $P(x), Q(x), R(x, y)$ satisfying the condition: For every real numbers a, b which satisfying $a^m - b^2 = 0$, we always have that $P(R(a, b)) = a$ and $Q(R(a, b)) = b$
- 3 Let an integer $n > 3$. Denote the set $T = \{1, 2, \dots, n\}$. A subset S of T is called *wanting set* if S has the property: There exists a positive integer c which is not greater than $\frac{n}{2}$ such that $|s_1 - s_2| \neq c$ for every pairs of arbitrary elements $s_1, s_2 \in S$. How many does a *wanting set* have at most are there ?



Vietnam
Team Selection Tests
2008



Day 2

- 1] Let m and n be positive integers. Prove that $6m|(2m+3)^n + 1$ if and only if $4m|3^n + 1$
- 2] Let k be a positive real number. Triangle ABC is acute and not isosceles, O is its circumcenter and AD, BE, CF are the internal bisectors. On the rays AD, BE, CF , respectively, let points L, M, N such that $\frac{AL}{AD} = \frac{BM}{BE} = \frac{CN}{CF} = k$. Denote $(O_1), (O_2), (O_3)$ be respectively the circle through L and touches \overline{OA} at A , the circle through M and touches \overline{OB} at B , the circle through N and touches \overline{OC} at C . 1) Prove that when $k = \frac{1}{2}$, three circles $(O_1), (O_2), (O_3)$ have exactly two common points, the centroid G of triangle ABC lies on that common chord of these circles. 2) Find all values of k such that three circles $(O_1), (O_2), (O_3)$ have exactly two common points
- 3] Consider the set $M = \{1, 2, \dots, 2008\}$. Paint every number in the set M with one of the three colors blue, yellow, red such that each color is utilized to paint at least one number. Define two sets:
 $S_1 = \{(x, y, z) \in M^3 \mid x, y, z \text{ have the same color and } 2008 \mid (x + y + z)\}$; $S_2 = \{(x, y, z) \in M^3 \mid x, y, z \text{ ha}$
Prove that $2|S_1| > |S_2|$ (where $|X|$ denotes the number of elements in a set X).